Implications of State-Contingent Mortgage Contracts in Housing Markets *

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Abstract

This paper quantifies the effect of government intervention in the U.S. housing market to determine whether it can explain the nonexistence of mortgage contracts that are contingent on house prices, which are found to be optimal in the mortgage design literature. In the model, contract and down payment choices, and the corresponding mortgage interest rates, are endogenous for heterogeneous households that are subject to idiosyncratic income and house price shocks, as well as to an aggregate house price shock. I find that the implicit subsidy to government-sponsored enterprises leads to the dominance of fixed-rate mortgages in the U.S. housing market, and in a world without government intervention, mortgage contracts that are contingent on house prices emerge endogenously. In this world, contingent contracts decrease the cyclicality of foreclosure rate by adjusting the value of debt during a housing crisis. However, they increase the average foreclosure rate in normal times due to endogenously decreasing the down payment of households, since contingent contracts are relatively cheaper for low down payment options in equilibrium.

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1 Introduction

Mortgage debt is the largest source of household debt in the U.S., so the terms of mortgage contracts are key not only for borrowers and lenders, but also for the economy as a whole. The Great Recession that followed the subprime mortgage crisis in the U.S. made it clear that a shock that affects the mortgage debt market can have sizable implications for the economy.

In the U.S., prior to Great Depression of the 1930s, mortgage contracts were mostly short term and included a bulk payment at the end of the loan period,—i.e., a balloon mortgage. In response to the Great Depression, government-sponsored enterprises (GSEs) were formed to ensure stability in the housing market. In 1933, GSEs set a precedent by introducing long-term, self-amortizing, fixed-rate mortgages as part of the New Deal, and these contracts have been the most popular form of mortgage financing since then[1].

The Great Recession that began in the late 2000s led to a debate about mortgage design and its implications during a crisis. Empirical research on the reasons for millions of foreclosures during the Great Recession emphasizes the importance of negative equity as a key trigger of default. Also, the literature on mortgage modification programs argues that even though principal reduction is an effective way to prevent mortgage default, there are barriers to adjusting the terms of a contract ex post. Thus, mortgage contracts that are contingent on house prices—which decrease the principal balance of the loan in the event of a house price decrease and prevent borrowers from falling into negative equity—are the most widely discussed proposal among unconventional mortgage contracts.

Part of the related literature focuses on the pricing of contingent contracts (CCs), based on financial formulas that ignore the interplay between contract prices and the optimal policy for borrowers. So, they find that contingent contracts should have a higher interest rate, since they transfer some of the house price risk from borrower to lender, but ignore that contingent contracts would also decrease the probability of default, and hence may have a lower interest rate in equilibrium. A few recent studies examine the implications of CCs in a general equilibrium setting, and broadly argue that CCs would improve welfare by decreasing foreclosures. This raises the following

[1] See Green and Wachter (2005) for more details about the evolution of mortgage markets in the U.S.

question: If so, why do contingent contracts not exist in practice? The conventional answer in the related literature is government intervention through either GSEs, tax policy, and/or banking regulations, but there has been no formal and quantitative analysis of this claim in the literature.

To conduct this analysis, I use a heterogeneous-agents life cycle model with idiosyncratic income and house price shocks, along with an aggregate house price shock parameterized for the study of the U.S. housing market, including the effect of government intervention. In the model, households that want to buy a house borrow from lenders if they do not have enough assets to be an outright owner. If they choose to borrow, they can borrow either with a standard fixed-rate, fully-amortizing mortgage contract (FRM) which has constant periodic payments or a CC that is also fixed rate and fully amortizing but periodic payments depend on the aggregate house price level. When the aggregate house price level decreases, periodic payments of a borrower are proportionally adjusted downward. If the house prices recover, payments go up again but never become higher than the original payment. Payment adjustments are not deferred to later periods, so they imply a principal reduction in the mortgage balance.

For each contract type, borrowers choose the level of down payment and then, depending on the contract type and down payment, a risk neutral lender sets the mortgage interest rate so that the expected return on the loan is equal to the cost of capital to issue the loan. Defaults are costly for the lender, since borrowers default when they have negative equity and also there is a foreclosure cost which the lender has to pay to sell the seized house. Thus, households with lower income and asset positions who have a high probability of default are subject to higher mortgage interest rates. Therefore, contract and down payment choices as well as mortgage interest rates, are all endogenous in the model.

In the U.S., lenders typically sell mortgage loans in the secondary market to investors to replenish their money and issue new mortgage loans. In the secondary market, lenders can sell their loans to GSEs or other private investors, but GSEs have a key advantage compared to private investors: The cost of capital for GSEs is lower due to an implicit government guarantee. Furthermore, GSEs only buy contracts that are consistent with their guidelines, which CCs do not conform to. Therefore, the competition between CCs and conforming loans—including FRMs—is tilted by the government in favor of conforming loans. I use the difference between the cost of capital for GSEs and private lenders to incorporate the effect of government in the housing market in my model.[2]

The contributions of this paper are twofold. First, I quantify the effect of government intervention in the U.S. housing market, and show that it is sufficient to explain the nonexistence of CCs in practice. Second, in a world without government intervention, I document the implications of CCs that adjust the value of debt in the event of an aggregate house price shock[3]. I show that CCs emerge endogenously in this world. Then, since CCs decrease the probability of default, their interest rate become lower for low down payment options. Households respond to these prices by borrowing with low down payment CCs at earlier periods of their life when they do not have much assets. As a result, foreclosure rate for CCs (0.6%) gets higher than the foreclosure rate for FRMs (0.3%) in normal times in which the contingency of CCs is not triggered. When the economy is hit with an aggregate house price shock, CCs adjust the value of debt, and the increase in the foreclosure rate becomes milder. While the foreclosure rate increases by 2.3% in the benchmark economy, and 2.0% in the data, it increases by 1.4% in the counterfactual world with contingent contracts.

2 Related Literature

The key feature of CCs is to prevent negative equity, and hence defaults and foreclosures. The empirical literature on mortgage default after the Great Recession shows that negative equity is an important trigger of mortgage default. This literature focuses on two models of mortgage default: the option value model and the double trigger model of default. The basic option value model of default implies that if the mortgage is underwater (i.e. the mortgage debt is higher than the value of the house), the borrower may prefer to walk away from this debt, even though there are certain costs (negative impact on credit score, social stigma, etc.). This is called strategic default. Second, the

[2] Passmore et al. (2005) derive a theoretical model which shows that the interest rate gap between conforming and nonconforming loans reflect the difference in cost of funding. They estimate GSE funding advantage to be 40 basis points. Ambrose and Warga (2002) estimate funding advantage of GSEs to be between 25-29 basis points over "AA" rated, between 43 and 47 basis points over "A" rated, and between 76 and 80 basis points over "BBB" rated banking sector bonds.

[3] CCs are contingent on aggregate but not idiosyncratic house prices to prevent moral hazard problem. A borrower can affect her own house price by deferring maintenance, but can not affect the aggregate house price level.

double trigger model of default implies that borrowers default because of they can not make their payments due to an adverse shock, In this case, however, negative equity is a necessary condition for default, since otherwise the borrower can choose to sell the house. Therefore, negative equity is a sufficient condition for default in the option value model and a necessary condition for default in the double trigger model of default.

Elul et al. (2010) study the interaction between two drivers of default.They emphasize the effect of liquidity shocks on mortgage default and show that this effect becomes more significant if combined with higher loan to value (LTV) ratios. Bhutta et al. (2010) study at what point borrowers strategically default, even if they can afford to pay. They find that 80% of defaults in their sample are the result of income shocks with negative equity. They also present empirical evidence for strategic default, but for very high LTV ratios. They claim that the median borrower does not strategically default until the LTV ratio rises to 162%, and half of the defaults are strategic if the LTV ratio is more than 150%. Guiso et al. (2013) study the determinants of strategic default by using survey data in which respondents are asked the following question: "Of the people you know who have defaulted on their mortgage, how many do you think walked away even if they could afford to pay the monthly mortgage?" They estimate that around 35% of the defaults were strategic in 2010. Lastly, Gerardi et al. (2015) study mortgage default using PSID data and find support for both the double trigger model and the option value model of default. Despite the discrepancies regarding at which point borrowers choose to default, it is well established that whether it is a necessary or a sufficient condition, negative equity is a key factor for mortgage default. My model is consistent with findings in the empirical literature, in the sense that borrowers mostly default if they are hit by both income and house price shocks. Also, they may choose to default without an income shock for very high LTV ratios. Thus, CCs that prevent negative equity are an effective way to prevent default and foreclosures.

The literature on CCs dates to much earlier than the Great Recession. Shiller (1994) discusses the importance of CCs in the housing market, together with the necessity of growth linked bonds, by claiming that contingent claims provide individuals or countries better hedging against macro risks. He develops this idea in subsequent papers by focusing on the pricing of these contracts (Shiller et al. (2013)) and the obstacles that hinder their introduction (Shiller (2014)). He argues that housing finance is in a primitive state and research or innovations in this area needs to be supported by the government, since this is a public good and innovators do not have any incentive to consider externalities.

Caplin (1997) also supports the idea of CCs with similar arguments and develops this idea in Caplin et al. (2007) and Caplin et al. (2008) by claiming that the tax benefits of debt contracts are a reason we do not observe innovative mortgage contracts in practice.

Another set of papers discusses the drawbacks of debt contracts and high leverage during and after the Great Recession period, which are a basis for CCs. The broad idea is that slow recovery is due to debt overhang and the deleveraging of households, and this slow recovery cannot be accelerated due to the zero lower bound. Debt contracts create rigidities, myopia, and contagion dangers. On the other hand, CCs adjust automatically in the event of a house price decrease, and thereby prevent a deleveraging process and enable better risk sharing (Crowe et al. (2013), Dynan et al. (2012), Turner (2012), Miles (2015)).

Mian and Sufi (2015) also support CCs in the housing market, following the lines in the literature, and provide substantial empirical evidence regarding the drawbacks of debt contracts for the macroeconomy. They argue that we do not observe CCs in practice due to government intervention in favor of debt contracts in the following ways: (i) the mortgage interest tax deduction, (ii) the effects of GSEs through buying conforming loans, and iii) banking regulations regarding risk weighted capital requirements. They also state that government should distort the market in favor of CCs, since these contracts prevent the externalities that arise due to foreclosures and would be leaning against the wind, since investors will be reluctant to lend if they think there might be a bubble in house prices.

With regard to studies of mortgage contract design, Piskorski and Tchistyi (2011) derive an optimal mortgage contract as a solution to a general dynamic contracting problem, and find that the optimal contract implies a decrease in principal balance in the case of a house price decrease. Similary, Eberly and Krishnamurthy (2014) propose a mortgage contract that reduces debt when house prices fall.

Few recent papers have examined the implications of CCs in an equilibrium housing model similar to this paper. Piskorski and Tchistyi (2017) find that widespread adoption of CCs has ambiguous effects on the homeownership rate and household welfare, which depend on the severity of recessions. Greenwald et al. (2017) find that CCs decrease the number of defaults and the consumption volatility of borrowers, but increase the consumption volatility of lenders. Similar to these two papers, I use an equilibrium model of housing; however the model I use and the question I answer are different.

In terms of modeling, the closest papers are Corbae and Quintin (2015), Chatterjee and Eyigungor (2015), Guler (2015) and Jeske et al. (2013). They all use an equilibrium model of housing with heterogenous agents and have endogenous mortgage interest rates such that risk neutral lenders make zero profit. Moreover, I introduce government intervention through government sponsored enterprises into the mortgage market in a way similar to Jeske et al. (2013).

3 Environment

Time is discrete. Households live for $\bar{J}$ periods and get retired at period J. There is a risk neutral lender that prices mortgage contracts endogenously. The economy is subject to three types of shocks: idiosyncratic income and house price shocks, and an aggregate house price shock. The expected lifetime utility of a household is given by:

$$\mathbb{E}_0\left[\sum_{j=1}^{J}\beta^{j-1}u_k(c_j)+\beta^{J+1}W(w_J,y_J)\right]$$

such that

$$u_k(c)=\frac{(\phi_k c)^{1-\sigma}}{1-\sigma}$$

where $\beta<1$ is the discount factor, j is the age of the household, c is consumption, and subscript k represents the current status of the household: inactive renter (d), active renter (r) or homeowner (h). The utility of the household depends on the current status, such that $u_h(c)>u_r(c)=u_d(c)$. Homeowners enjoy a utility premium over active and inactive renters. This premium is the main reason for purchasing a house in the model, and captures any benefit of owning a house, rather than renting that are not explicitly modeled here.

Households are ex ante identical and start their lifecycle as renters. An active renter can remain an active renter or become a homeowner by purchasing a house. She can either become an outright owner by paying the purchase price with her own assets or borrow from the lender. If she chooses to borrow, she will select from a set of feasible contracts that the lender offers. She chooses the optimal contract type, down payment and corresponding interest rate. Available contract types are standard fully amortizing fixed rate mortgages (FRMs) and mortgage contracts contingent (CCs) on house prices.

Mortgage contracts contingent on house prices are studied extensively in the literature, and even though the key feature is always to prevent negative equity, details of the contract vary across different papers. Among the CCs discussed in the literature, the closest to the one I study is proposed by Mian and Sufi (2015). Borrowers make a fixed payment every period for their mortgage debt, and this amount is proportionally adjusted downward if house prices decrease. If house prices increase, payments go up again but never become higher than the original payment. Payment adjustments are not deferred to later periods, so they imply a principal reduction in the mortgage balance. I do not model any shared appreciation at the end of the life of the contract.

A homeowner can remain a homeowner by making at least the periodic mortgage payment, become a renter by selling the house, or become an inactive renter by defaulting on the mortgage debt. If the homeowner chooses to sell the house, she will pay a selling cost θ_h. If the homeowner choose to default, the lender seizes the house and sells it after paying foreclosure cost θ_l. θ_l represents the foreclosure costs for the lender, such as legal costs, deferred maintenance costs, or foreclosure delays. This is a key parameter that makes CCs appealing for both borrowers and lenders. It represents a lost value for both, and they can share this otherwise lost value by using a CC that prevents default.

Inactive renters who have a default flag in their history are excluded from purchasing a house for a certain number of periods, and an inactive renter can be an active renter with a certain probability δ. Being excluded from purchasing a house is the only penalty for default in the model. $W(w_J, y_J)$ is the deterministic utility of the household after retirement and depends on last-period income and wealth. There is no uncertainty for retired households, and their problem can be solved analytically.

The logarithm of the income process follows the equation below:

$$
\begin{aligned}
log\left(y(j, z_j)\right) &= f(j) + z_j \\
z_j &= \rho_z z_{j-1} + e_j
\end{aligned}
$$

where $y(j, z_j)$ is the income at age j and with shock z_j. Therefore, log income up to retirement is composed of an age-dependent deterministic pattern $f(j)$ and a shock z_j that follows an AR(1) process. The persistency of the AR(1) process is ρ_z, and e_j is an i.i.d. Gaussian innovation with 0 mean and standard deviation σ_e. Idiosyncratic house prices also follow an AR(1) process with a persistency ρ_p, mean level of $\bar{p}$, and i.i.d. Gaussian innovation ϵ_j. ϵ_j is normally distributed with 0 mean and a standard deviation of σ_ϵ.

$$log\, p_{j+1} = \bar{p}(1-\rho_p) + \rho_p\, log\, p_j + \epsilon_j$$

House prices are also subject to aggregate shocks. Aggregate house prices follow a Markov process defined on the state space $Q \equiv \{q_{s_1}, .., q_{s_n}\}$, where $s_1, ..., s_n$ are different states of the world. Aggregate house prices are normalized such that $E_s[q_s] = 1$. The state space and the transition matrix will be calibrated to match long-run house price dynamics in the U.S., and mortgage contracts are contingent on aggregate house prices but not on idiosyncratic shocks. This is the convention followed in most of the related literature to solve the moral hazard problem of CCs. A borrower can affect her own house price by deferring maintenance and consequently decrease the mortgage payments, but can not affect the aggregate house prices. Also, while assessing the value of an individual house is costly, aggregate house price indices are already available.

Households facing the idiosyncratic and aggregate shocks described above can save at the risk-free interest rate r_f to smooth consumption. They can choose to borrow from lenders to purchase a house, but no unsecured borrowing is allowed. Also, households with a mortgage debt cannot hold assets at the same time, but in the case of a positive income shock, households can choose to pay more than the periodic mortgage payment without penalty.

The lender is risk neutral and sets the mortgage prices so that the expected return on loans is equal to the opportunity cost of the funds required to issue those loans. The lender can sell the loans in the secondary market to either private investors or GSEs, which ask for a return equal to the opportunity cost of their own funds. The cost of capital is r_f for GSEs that buy only FRMs in the secondary market and $r_f + \tau$ for private investors buying CCs. τ represents the difference between the cost of capital for GSEs and private investors due to an implicit government guarantee for GSEs. GSEs' funding advantage, τ, is estimated in the empirical real estate literature; and I am going to use this value but will also show the sensitivity of my results to different values of τ.

A risk neutral lender offers two type of contracts with various down payment options to households: an FRM or a CC with a maturity up to retirement age, J. Households are expected to pay their mortgage debt before retirement, so periodic payments are computed so that the discounted present value of periodic payments up to retirement is equal to the value of the debt. Although the maturity of the contract is deterministic ex ante, borrowers have the option to prepay the loan without penalty, so the maturity of the contract is also endogenous. For each contract type and downpayment level, an interest rate will pin down the next period's debt and the probability of default for the household. The lender will choose the interest rate so that the expected return on capital is equal to the cost of capital for the lender. In this setting, households with low income and assets, who have a high probability of default, will be subject to a higher risk premium and mortgage interest rates.

Timing in each period is as follows. At the beginning of each period, all three shocks are realized so that households learn their income level, house price, and aggregate status. Inactive renters make their consumption and saving choices, active renters decide whether to buy a house, and homeowners decide whether to keep, sell or default. At the end of the period, income is received and consumption takes place. Since housing status is decided at the beginning of the period, for instance, a homeowner who chooses to default at the age of j gets utility of $u_d(c_j)$ for that period.

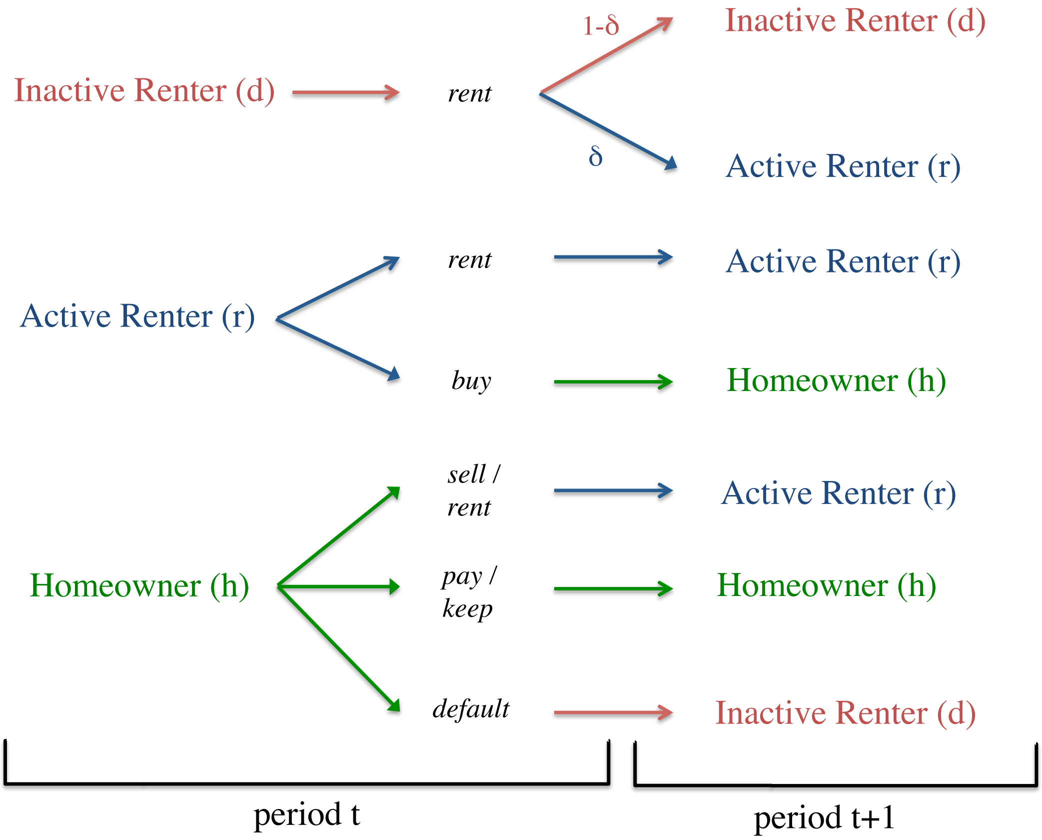
4 Decision Problems

This section presents the decision problems solved by three types of household and a lender's problem that equalizes the opportunity cost and return of funding for investors.

4.1 Households

Inactive renter chooses only the consumption and saving allocations, and rents a house for the current period. She starts the next period either as an active renter with probability δ or an inactive renter with probability $1-\delta$. So the value function of an inactive renter is:

Figure 1: Housing status choices
This figure shows the housing status choices of hoseholds. All households start their lifecycle as active renters and can then go through each status during lifetime. Inactive renters can only choose to rent for the current period but can then be an active renter with probability δ. For active renters and homeowners, next-period status depends on the policy function at the current period.



$$V_j^d(a, z; s) = \max_{c,a' \geq 0} \Big\{ u_d(c) + \beta \mathbb{E}\big[\delta V_{j+1}^r(a', z'; s') + (1-\delta) V_{j+1}^d(a', z'; s')\big] \Big\}$$

subject to

$$c + a'/(1 + r_f) = y(j, z) + a$$

where d indicates that the household is an inactive renter and j is the age of the household. The value function depends on current assets, household income and aggregate state of the economy. In the next period, the inactive renter becomes an active renter with probability δ, and the value function of the active renter is denoted by V^r. In the budget constraint, the next period's asset level is $a' \geq 0$, since no unsecured borrowing is allowed in the model, so r_f denotes the risk-free interest rate at which all savings in the model are accumulated.

Active renter can choose to remain an active renter or purchase a house. Therefore, the value function of an active renter is equal to the maximum of these two options:

$$V^r = \max\left\{V^{rr}, V^{rh}\right\}.$$

where V^{rr} denotes an active renter who chooses to stay an active renter and V^{rh} denotes an active renter who chooses to purchase a house.

An active renter who chooses to stay an active renter must choose consumption and saving allocations. Similar to the problem of the inactive renter, she chooses a saving level $a^{'} \geq 0$, and her savings are subject to interest rate r_f. The value function is denoted by:

$$V_j^{rr}(a, z; s) = \max_{c, a^{'} \geq 0} \left\{u_r(c) + \beta \mathbb{E} V_{j+1}^r\left(a^{'}, z^{'}; s^{'}\right)\right\}$$

subject to

$$c + a^{'}/(1 + r_f) = y(j, z) + a$$

Alternatively, an active renter can choose to buy a house, and the value function is denoted by V^{rh}. In this case, the household chooses consumption level c, contract type γ', and downpayment. Depending on the contract type and the downpayment, the next period's debt level $a^{'}$ and mortgage interest rate r_m are pinned down so that the lenders make zero profit. The value function is as follows:

$$V_j^{rh}(a,z;s) = \max_{c\geq 0,(a',r_m,\gamma',s)\in\Upsilon(a,z;s)} \left\{ u_h(c) + \beta \mathbb{E} V_{j+1}^h \left(a^{'}, z^{'}, p_h^{'}, r_m, \gamma^{'}, s; s^{'} \right) \right\}$$

subject to

$$c + a^{'}/(1+r_m) + \bar{p_h} q_s = y(j,z) + a$$

where $\Upsilon(a,z;s)$ refers to the set of feasible contracts, depending on the state variables of the value function. Therefore each household with different observables has access to a different set of mortgage contracts. A contract is defined by four variables: the next period's level of debt $a^{'}$, mortgage interest rate r_m, contract type $\gamma^{'}$, and the state of the economy in which the contract is issued, s. Compared to a renter, a homeowner has a house and a mortgage contract. Thus, state variables for mortgage contract and house price are added to the set of state variables for a homeowner. Construction of the feasible set of contracts will be further explained in the lender's problem.

The homeowner's value function is a nested formulation that encompasses all possible statuses for a homeowner but the state variables I need to keep track of depend on the status of the homeowner. Contract type $\gamma^{'}$ takes three values that denote whether the homeowner is an outright owner, borrower with a CC or borrower with an FRM. If the active renter buys a house with her own assets and becomes an outright owner, $a^{'} \geq 0$ and $r_m = r_f$ since savings are subject to risk-free interest rate. If the active renter purchases a house by borrowing from lenders, $a^{'} < 0$ and the borrower cannot hold any other asset. If the borrower holds an FRM, I do not need to keep track of s, which denotes the aggregate state in which the contract is issued. If the borrower holds a CC, since payment adjustments are based on the state in which the contract is issued, s becomes a state variable. While $\bar{p_h}$ is the average house price and also the purchase price of a house at the steady state, $p_h^{'}$ is the house price next period.

Homeowner has three options: keep, sell, or default if there is any mortgage debt. Therefore, the value of homeownership is the maximum value of these options.

$$V^h = \max\{V^{hh}, V^{hr}, V^{hd}\}$$

where V^{hh} denotes the homeowner who keeps the house, V^{hr} denotes the homeowner who sells the house and V^{rd} denotes the homeowner who defaults on mortgage debt.

The value function of a homeowner depends on the level of assets (debt if $a < 0$), income, house price, mortgage interest rate, and contract type, current aggregate state, and the aggregate state in which the homeowner purchased a house. The homeowner who defaults on the mortgage debt chooses consumption and saving allocations, and the value function is denoted by:

$$V_j^{hd}(a, z, p_h, r_m, \gamma, s_o; s) = \max_{c,a' \geq 0} \left\{ u_d(c) + \beta \mathbb{E} \left[\delta V_{j+1}^r \left(a', z' \right) + (1-\delta) V_{j+1}^d \left(a', z' \right) \right] \right\}$$

subject to

$$c + a'/(1+r_f) = y(j,z) + max\{(1-\theta_l)p_h q_s + a, 0\}$$

Following default, similar to an inactive renter, a homeowner can become an active renter with probability δ or an inactive renter with probability $1-\delta$. In the case of a default, the lender seizes the house and sells it for $(1-\theta_l)p_h$, where θ_l represents the foreclosure costs. If this amount is more than the mortgage balance, the lender pays back the remaining amount to borrower; this never happens in equilibrium. If the loan amount is less than the house value, the homeowner, who is subject to a negative income shock and cannot make the periodic payments, chooses to sell the house. This implies that negative equity is a necessary condition for mortgage default in the model, and this implication is consistent with the empirical literature on mortgage default[4].

A homeowner can also sell the house and become a renter in the next period. In this case, the homeowner only chooses consumption and asset allocations and pays back the debt, if any. The value function is denoted by:

$$V_j^{hr}(a, z, p_h, r_m, \gamma, s_o; s) = \max_{c,a' \geq 0} \left\{ u_r(c) + \beta \mathbb{E} V_{j+1}^r \left(a', z' \right) \right\}$$

[4]Households choose to default only if $-a \geq (1-\theta_h)p_h q_s$.

subject to

$$c + a'/(1 + r_f) = y(j, z) + a + p_h q_s (1 - \theta_h)$$

where θ_h is the cost of selling a house for a homeowner. In this case, the homeowner sells the house, pays back the debt a (if any), and saves assets equal to $a^{'} \geq 0$. The savings are subject to risk-free interest rate r_f.

The last option of a homeowner is to keep the house. If there is any mortgage debt, the homeowner must pay at least the periodic payment, but can choose to pay more. The value function is denoted by V^{hh}. Since the value function for each debt status is subject to different budget constraints, instead of the nested formulation, I will present each case separately. Let $\gamma = 1$ denote an outright owner, $\gamma = 2$ denote an FRM holder, and $\gamma = 3$ denote a CC holder. Then, the value function for an outright owner is as follows:

$$V_j^{hh}(a, z, p_h, r_f, 1, s_o; s) = \max_{c, a^{'} \geq 0} \left\{ u_h(c) + \beta \mathbb{E} V_{j+1}^h \left(a^{'}, z^{'}, p_h^{'}, r_f, 1, s_0; s^{'} \right) \right\}$$

subject to

$$c + a^{'}/(1 + r_f) = y(j, z) + a$$

since an outright owner saves at the risk-free interest rate. Note that state q_0 is not relevant for the problem of an outright owner. For a homeowner with an FRM,

$$V_j^{hh}(a, z, p_h, r_m, 2, s_o; s) = \max_{c, a^{'} \geq \underline{a}} \left\{ u_o(c) + \beta \mathbb{E} V_{j+1}^h \left(a^{'}, z^{'}, p_h^{'}, \tilde{r_m}, \gamma^{'}, s_o; s \right) \right\}$$

subject to

$$c + a^{'}/(1 + \tilde{r_m}) = y(j, z) + a$$
$$\underline{a} = (a + m(a, r_m))(1 + r_m)$$

$$a^{'} = \begin{cases} < 0 & \tilde{r}_m = r_m \text{ and } \gamma^{'} = 2 \\ \geq 0 & \tilde{r}_m = r_f \text{ and } \gamma^{'} = 1 \end{cases}$$

A homeowner with an FRM has to pay at least the periodic payment, but then can choose to keep a positive mortgage balance (negative assets) or become an outright owner. If the level of assets for the next period is negative, the mortgage balance is subject to interest rate r_m and $\gamma^{'} = 2$. Otherwise, the household saves at the risk free rate r_f and becomes an outright owner, so $\gamma^{'} = 1$. The household must make at least the periodic payment $m(a, r_m)$ due to conditions $a^{'} \geq \underline{a}$ and $\underline{a} = (a + m(a, r_m))(1 + r_m)$. Periodic payments are such that the present discounted value of payment streams up to retirement is equal to debt; this will be further explained in the lender's problem. Note that the state s_0 is again not relevant for the problem of an FRM holder. Lastly, for a homeowner with a CC,

$$V_j^{hh}\left(a, z, p_h, r_m, 3, s_o; s\right) = \max_{c, a^{'} \geq \underline{a}} \left\{ u_o(c) + \beta \mathbb{E} V_{j+1}^{h}\left(a^{'}, z^{'}, p_h^{'}, \tilde{r_m}, \gamma^{'}, s_0; s\right) \right\}$$

subject to

$$c + a^{'}/(1 + \tilde{r_m}) = y(j, z) + a + m(a, r_m)(1 - \alpha)$$
$$\underline{a} = (a + m(a, r_m))(1 + r_m); \quad \alpha = min\left(\frac{q_s}{q_{s_0}}, 1\right)$$

$$a^{'} = \begin{cases} < 0 & \tilde{r}_m = r_m \text{ and } \gamma^{'} = 3 \\ \geq 0 & \tilde{r}_m = r_f \text{ and } \gamma^{'} = 1 \end{cases}$$

This case is similar to the problem of a household holding an FRM; the only difference is the extra term in the budget constraint, $m(a, r_m)(1 - \alpha)$. This implies that if the aggregate house price is below the level at which the mortgage contract was issued, the borrower has to pay less and the principal balance is reduced. For instance, if aggregate house price decrease by 20% and stay at that level up to the end of the contract period, a homeowner who does not choose to prepay would pay 20% less, both periodically and in total. This formulation is not standard in the literature, since most of the models that study CCs do not have a prepayment option. In the related literature, typically debt is adjusted proportional to change in house prices. This formulation is similar to that convention, but takes into account the implications of the prepayment option. If I multiply debt with α in my model, capable households pay as much debt as possible if aggregate house price is below the initial condition so that they can be an outright owner before house prices recover. This, in turn, leads to an unrealistic implication that the mortgage rates of CCs are higher for high-income individuals.

4.2 The Lender

There is a risk neutral lender with deep pockets who makes zero profit in the long run. Idiosyncratic shocks wash out due to the law of large numbers, but due to aggregate shocks, the lender can have positive or negative profit each period. The lender sets the interest rate for each contract to ensure zero profit in the long run. Therefore, I assume that the lender has deep pockets and is capable of bearing the losses in bad states. The lender offers households a set of feasible contracts $(a', r_m, \gamma^{'}, s) \in \Upsilon(a, z, s)$ that includes certain combinations of contract type, debt level, and interest rate. A contract type and down payment pin down the level of debt for the current period. Then for any interest rate, next period's level of debt can be found. Since the model is solved backward, the lender knows the probability of default for each level of debt and interest rate. The lender sets the interest rate so that the expected return on capital is equal to the cost of funding. Borrowers can pay back the mortgage debt up to their retirement, so the borrower's age determines the maturity of the contract. Periodic payments are computed such that the present discounted value of lifetime payments is equal to the value of the debt. Given the interest rate and the level of debt, periodic payments $m(a, r_m)$ at the age of j are computed as follows:

$$
\begin{aligned}
d = m + \frac{m}{1+r_m} + \frac{m}{(1+r_m)^2} + ... + \frac{m}{(1+r_m)^{J_r - j}} \\
\Rightarrow m(d, r_m) = \frac{1 - 1/(1+r_m)}{1 - 1/(1+r_m)^{J_r - j + 1}} d
\end{aligned}
$$

The value of a homeowner's debt depends on the observables of the homeowner and is denoted by V_j^l. In the following equation, the most important departure from the literature is to assume that the cost of capital for issuing CCs is higher than the cost of capital for issuing FRMs. This is to reflect that the cost of capital for GSEs is lower than the cost of capital for private lenders in the data. The value function of debt for the period of borrowing is:

$$V_j^l(a, z, p_h, r_m, \gamma, s; s) = m(a, r_m) + \frac{1}{1 + r_f + \kappa + \mathbb{1}_{k=3}\tau} \mathbb{E} V_{j+1}^l(a^{'}, z^{'}, p_h^{'}, r_m, \gamma, s; s^{'})$$

and r_m is such that

$$V_j^l(a, z, p_h, r_m, \gamma, s; s) = -a$$

Note that the first state in the value of debt does not represent the assets of the household. The first state variable a in V_j^l denotes the current level of debt, which is the difference between the purchase price of the house and the downpayment. Then a household who takes a mortgage loan of a makes a periodic payment of $m(a, r_m)$ at the current period, and the next period's debt level $a^{'} = (1 + r_m)(a + m(a, r_m))$ is pinned down, so the household cannot pay more than the periodic payment at the period of borrowing. Also, in the period that the contract is issued, $s_0 = s$. κ is the service cost of lending and will be jointly calibrated in the model. For the periods after origination of the contract, the value of debt is equal to:

$$V_j^l(a, z, p_h, r_m, \gamma, s_o; s) = \begin{cases} -a + \frac{a^{'}}{(1+r_m)} - \overbrace{[\mathbb{1}_{k=3} m(...)(1 - \alpha)]}^{\textit{payment adjustment}} + \frac{1}{1+r_f+\kappa+\mathbb{1}_{k=3}\tau} \mathbb{E} V_{j+1}^l(...) & \text{stay} \\ -a & \text{sell} \\ \underbrace{\min\left[p_h q_s(1 - \theta_l), -a\right]}_{p_h q_s(1-\theta_l)\ \textit{in equilibrium}} & \text{default} \end{cases}$$

A homeowner with mortgage debt has three options. If the homeowner chooses to keep the house, she makes the payment for that period $-a + a^{'}/(1 + r_m)$, which may be bigger than the periodic payment. Then the next period's value of debt is discounted with the cost of capital. If the mortgage contract is CC, the payment is adjusted proportional to house price change. If the homeowner chooses to sell the house, the lender gets back the debt. If the homeowner chooses to default, the lender seizes the house and sells it for $p_h q_s(1-\theta_l)$ where θ_l is the foreclosure cost. If this amount is bigger than the level of debt, the lenders pays back the remaining amount to the borrower but in equilibrium borrowers do not default if $p_h q_s(1-\theta_l) \geq -a$.

5 Parameter Selection and Calibration

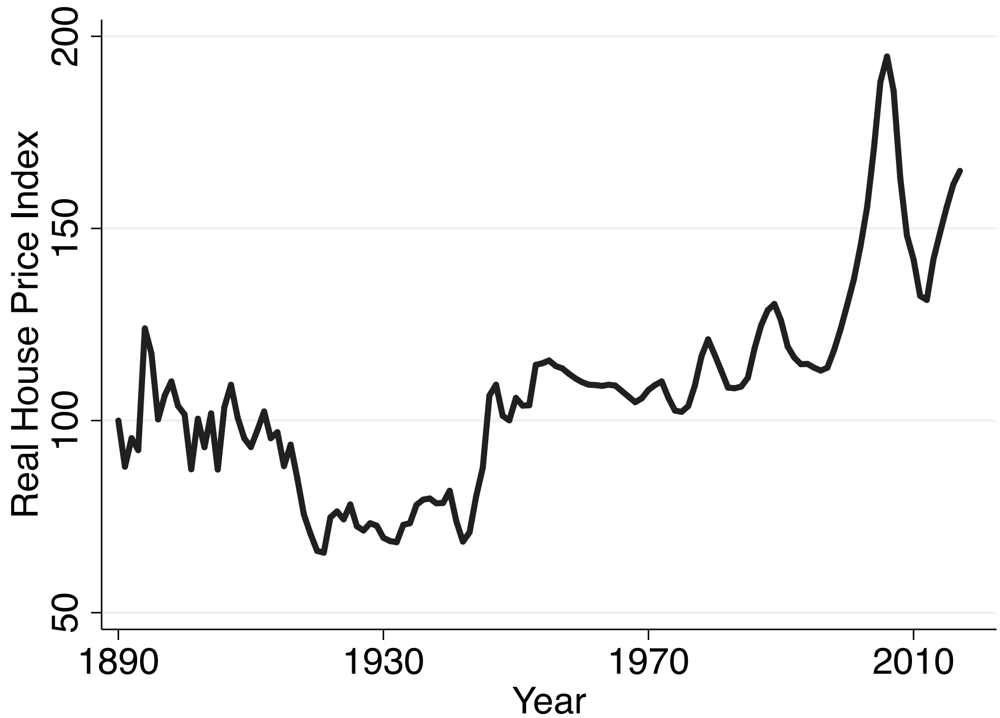
Table 1: Parameters selected independently

Description	Parameter	Value
Persistence of income	ρ_z	0.97
Std of income shock	σ_z	0.13
Persistence of idio. house price	ρ_p	0.96
Std of idio. house price shock	σ_p	0.1
Mean house price to income	$\bar{p_h}$	3.45
Selling cost—household	θ_h	0.07
Selling cost—lender	θ_l	0.27
Risk aversion	σ	2
Discount rate	β	0.96
Risk-free interest rate	r_f	0.04
Difference in cost of capital	τ	45bp

The model period is one year, and households live for 36 years up to retirement. Among the many studies that estimate the log earnings for the U.S., I follow Storesletten et al. (2004) and set the persistency of the income process to 0.97 and the standard deviation of income shocks to 0.13. For idiosyncratic house prices, I follow Nagaraja et al. (2011) and set the persistency of house price shocks to 0.96 and the standard deviation of shocks to 0.1. I set $\bar{p_h}$ to 3.45, consistent with U.S. Census median house price data for the period 1990-2000. For aggregate house price shocks, I follow Corbae and Quintin (2015) and use the real house price index since 1890 to calibrate a Markov chain. To do the calibration, I use the empirical calibration method outlined in Civale et al. (2016).

Regarding the selling cost of a house for a homeowner, I use the estimate of Gruber and Martin (2003). They find that the median homeowner pays 7% of the house's value

Figure 2: U.S. real house price index. Source: Shiller (2000); updated data is available at `http://www.econ.yale.edu/~shiller/data.htm`



to sell it. For the selling cost of a lender for a foreclosed house, I use the estimate of Campbell et al. (2011) and set θ_l to 0.27. I set the risk aversion coefficient for the CRRA utility function to 2, discount factor β to 0.96, and risk-free interest rate r_f to 0.04. Finally, τ is set to be 45 basis points, which is the average estimate of the difference between return on GSE debt and A-rated banking sector bonds in Ambrose and Warga (2002).

The rest of the parameters are calibrated jointly. The utility premium of homeownership ϕ_h, probability that an inactive renter will be an active renter δ, and service cost of lender κ are calibrated to match the homeownership rate, foreclosure rate, and mortgage premium in the data. The homeownership rate is the average homeownership rate in U.S. Census data for the period 1990-2000. For the foreclosure rate, I use data for foreclosure starts from the Mortgage Bankers Association National Delinquency Survey for the period 1990-2000. Then, I assume that the foreclosure rate is half of the foreclosure starts, based on the evidence in Herkenhoff and Ohanian (2015). Foreclosure starts between 1990 and 2000 is roughly around 3.5% of all mortgages in a quarter. Thus, I find an average annual foreclosure rate of 0.68% for the relevant period. Finally, the mortgage premium is defined as the difference between the mortgage rate for a 30-year FRM and the 10 year constant maturity treasury rate. The mortgage interest rate is from the Monthly Interest Rate Survey of the Federal Housing Finance Agency for the period of 1990-2000.

6 Results

In this section, I will first discuss model fit and the effect of government intervention by comparing the benchmark model with some counterfactuals. I then demonstrate the implications of CCs in a counterfactual world, both at the steady state and in the case of an aggregate house price shock.

6.1 Model fit

There are three parameters ϕ_h, δ, and κ that I jointly calibrate to match the homeownership rate, foreclosure rate, and mortgage premium in the data. In table 2, the first three moments are matched by construction; untargeted moments, LTV ratio and average down payment are also close to their data counterparts.

Table 2: Parameters selected jointly
This figure compares five moments of the model with their data counterparts. The first three targeted moments are matched by construction. Loan to value and down payment ratios are also close to data, although they are not targeted. Data moments are for the period 1990-2000. The homeownership rate is from the U.S. Census, the foreclosure rate is from the Mortgage Bankers Association, the mortgage premium and the down payment are from the Monthly Interest Rate Survey of the Federal Housing Finance Agency, and lastly the LTV is from flow of funds. Down payment is the ratio of initial payment to value of the loan at the origination while LTV ratio is for all the existing mortgage contracts. This is why LTV $\neq$ 1-Down payment.

Targeted statistics	Data(%)	Model(%)	Parameter	Value
Homeownership rate	65	65	Utility prem. ϕ_h	0.49
Foreclosure rate	0.68	0.69	Prob. of active δ	0.27
Mortgage prem.	1.55	1.46	Service cost κ	98bp
Untargeted statistics				
LTV	58	58		
Down payment	20	22		

Among the three jointly calibrated parameters, the utility premium of being a homeowner increases the homeownership rate, but increasing the homeownership rate also increases the foreclosure rate and mortgage premium, since risky households that would not take out a loan in the case of a low ϕ_h borrow, and then some of these borrowers default. The average mortgage premium increases since lenders charge a higher interest rate for the risky borrowers. The probability of being an active renter δ is the only penalty for default. A lower δ prevents inactive renters from becoming a homeowner for a longer period, and thus implies a higher penalty. Therefore, lower δ leads to a lower foreclosure rate in the economy. Service cost κ increases the mortgage premium and has two balancing effects on the foreclosure rate. A higher interest rate increases the periodic payments and the probability of default, but on the other hand, a higher interest rate leads active renters to make higher down payments and thus may also decrease the probability of default.

6.2 Government intervention and counterfactuals

The mortgage contract literature on CCs argues that we do not observe CCs in the data due to the government intervention in the mortgage market. In this paper, I quantify the effect of government intervention by using different costs of funding for CCs and FRMs, since the former one does not conform to the guidelines of GSEs and therefore can only be sold to private investors in the secondary market; and also cost of funding is lower for GSEs due to an implicit government guarantee. I then analyze two counterfactuals in which the cost of funding is the same for both contracts. In the first counterfactual (CF1), I set the cost of capital to $r_f + \tau$ for both types of contracts; this is a proxy for a case in which there is no government intervention in the housing market. In the second counterfactual (CF2), I set the cost of capital for both CCs and FRMs to r_f to represent a case in which GSEs buy both FRMs and CCs in the secondary market.

The share of each household status through lifetime is given in table 3. Blue rows show the share of each possible status for a household: an inactive renter, active renter or homeowner. The shares of these three statuses add up to 100. Light gray rows show the three possible statuses of a homeowner: an outright owner, CC holder or FRM holder. The sum of shares for these three rows should add up to the homeownership

Table 3: Shares of each household status and the optimal policy

This table shows the share of each household status at the steady state for the benchmark economy, CF1 and CF2. A homeowner can be an outright owner, a CC holder or an FRM holder; I report the share of each status and the share of policy functions for each status. An inactive renter does not have a choice for the next period, so it is not reported. An active renter can stay as a renter or can purchase a house by becoming an outright owner, a CC holder or an FRM holder. An outright owner can sell the house and become a renter or keep the house. A loan holder can sell the house and become a renter or keep the house with or without the loan or default and become an inactive renter.

		Benchmark			CF1			CF2		
		Share	Income	Wealth	Share	Income	Wealth	Share	Income	Wealth
All households		100	1.59	4.53	100	1.59	4.53	100	1.59	4.50
Inactive renter		0.3	0.62	0.23	0.2	0.61	0.23	0.7	0.67	0.27
Active renter		34.6	0.53	0.20	36.3	0.55	0.20	34.1	0.53	0.19
	Rent	90.2	0.42	0.19	91.0	0.44	0.19	90.0	0.42	0.19
	CC	0.0	-	-	5.3	1.82	0.19	6.2	1.63	0.10
	FRM	9.7	1.53	0.24	3.7	1.39	0.43	3.8	1.29	0.43
	Outright	0.003	4.50	1.75	0.02	4.66	1.34	0.004	4.21	2.57
Homeowner		65.0	2.15	6.85	63.5	2.18	7.01	65.2	2.15	6.80
Outright		46.6	2.42	9.12	46.5	2.42	9.12	46.3	2.43	9.13
	Sell	0.7	0.25	2.59	0.7	0.25	2.59	0.7	0.25	2.59
	Keep	99.3	2.44	9.16	99.3	2.44	9.17	99.3	2.44	9.17
CC		0.0	-	-	9.4	1.58	1.20	11.2	1.53	1.01
	Sell	-	-	-	4.5	0.38	1.17	3.6	0.36	1.21
	Keep	-	-	-	94.9	1.64	1.21	94.5	1.60	1.03
	Default	-	-	-	0.6	0.32	-0.14	1.9	0.38	-0.09
FRM		18.5	1.47	1.15	7.6	1.43	1.27	7.7	1.39	1.19
	Sell	4.6	0.33	1.19	5.1	0.31	1.30	5.0	0.30	1.26
	Keep	94.7	1.53	1.15	94.7	1.50	1.27	94.6	1.46	1.20
	Default	0.7	0.31	-0.11	0.3	0.25	-0.02	0.4	0.24	-0.04

rate. The rest of the rows represent shares of individuals for each housing choice, given the current status of the household and normalized so that the sum of the shares of all possible choices for an individual is 100. For instance, the share of active renters who choose to buy a house using an FRM contract is equal to 9.7% of all active renters—not the entire economy—in the benchmark case. Also, the share of active renters who become outright owner is very low; however households with mortgage loans pay all of their debts in time and become outright owners in later periods of their lives so the share of outright owners is 46.6% in the benchmark economy.

Average income in the economy is 1.59 and is constant, since it is exogenous for the economy. However, the average wealth depends on household choices and is lower in CF2. Comparing the three economies and the wealth of three types of households, the share of inactive renters is very low and the wealth of active renters is almost the same across the three economies. Homeowners' wealth increases as the opportunity cost of lender increases so that wealth for homeowners is equal to 6.80, 6.85 and 7.01 for CF2, the benchmark, and CF1, respectively. Thus, total wealth in CF2 is the lowest since the cost of funding and hence the mortgage interest rates is lowest in CF2. In CF1, the cost of funding is higher compared to the benchmark economy, and thus the wealth of homeowners is higher; however the homeownership rate is lower. Thus, total wealth in the benchmark economy and CF1 are the same.

The share of inactive renters is highest in CF2 and lowest in CF1. This is consistent with the default rates in the lower rows of the table. The default rate is 0.7 for FRM holders, which is the only contract in the benchmark economy. The default rate for CCs and FRMs is 1.9 and 0.4, respectively, in CF2 and 0.6 and 0.3 in CF1. Moving from the benchmark economy to CF2, the share of inactive renters or default rates increase, since CCs have a higher default rate at the steady state. Although this may seem surprising at first, CCs have a higher interest rate (see Table 4), since they provide insurance against aggregate shocks but if aggregate shocks do not hit, individuals who pay higher periodic payments due to higher interest rates become more likely to default. On the other hand, default rates are the lowest in CF1 since households save more and make bigger down payments (see Table 4) due to higher cost of capital.

A key result of this paper is about the optimal decision of active renters. In the benchmark economy, none of the active renters use a CC due to the difference in the cost of capital. It is claimed in the literature that government intervention leads to the nonexistence of CCs, and these results verify this claim by quantifying government intervention using the difference in the cost of capital for GSEs and private lenders. CCs exist in both counterfactuals that the cost of capital is the same for both contracts.

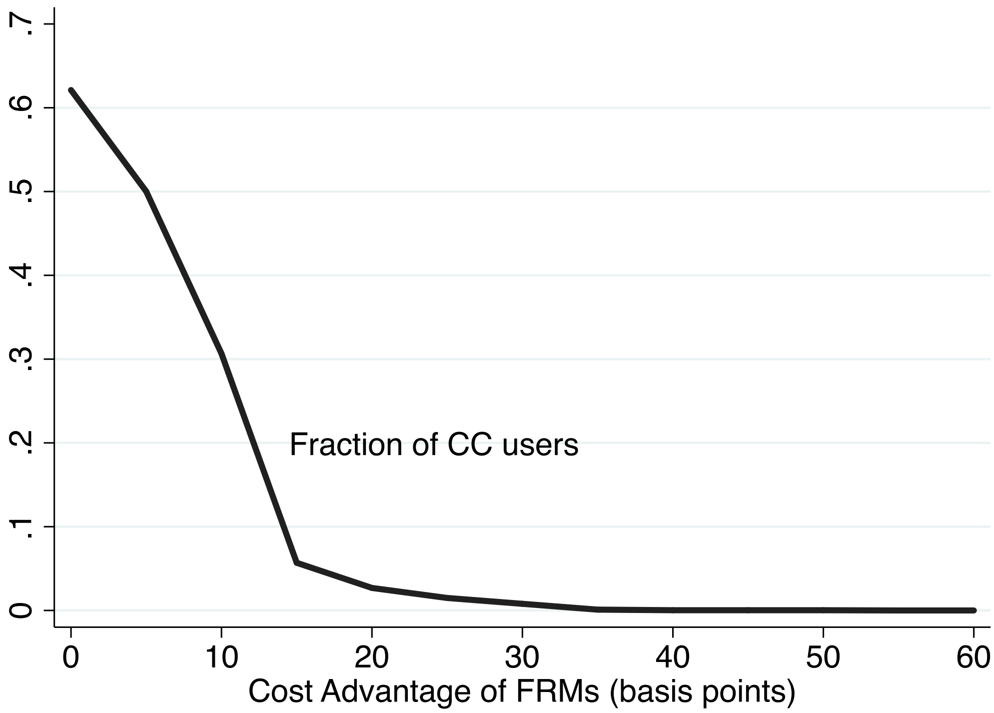
Table 4: Moments of counterfactuals

	Benchmark FRM $\rightarrow r$ CC $\rightarrow (r+\tau)$	CF1 FRM $\rightarrow r+\tau$ CC $\rightarrow r+\tau$	CF2 FRM $\rightarrow r$ CC $\rightarrow r$
Av. premium	1.46	2.08	1.96
CC premium	-	2.36	2.36
FRM premium	1.46	1.69	1.31
Av. downpayment	22.2	25.2	20.5
CC downpayment	-	26.3	19.7
FRM downpayment	22.2	23.7	21.7

Comparing the average income and wealth of CC holders demonstrate who would be interested in CCs in this counterfactual world. Caplin et al. (2008) provide survey evidence for interest in another form of mortgage financing contingent on house prices: shared appreciation mortgages (SAMs). Even though SAMs differ from the contracts I study, a key feature is that they also prevent borrowers from falling into negative equity. Caplin et al.'s survey results show that young, low-income, low-wealth households expecting higher income growth would be interested in SAMs. In Table 3, comparing the income and wealth of active renters choosing to buy a house with a CC or an FRM, or comparing homeowners holding a CC or an FRM both suggest the following: High-income low-wealth individuals are more interested in CCs. I will also show later that mostly younger individuals choose to borrow with CCs. My results are therefore mostly consistent with the survey findings of Caplin et al., except I find that high-income individuals are more interested in CCs.

Income level affects the policy function of a household in any status in a reasonable fashion. Among active renters, households that remain renter have the lowest income, and ones that choose to be outright owner have the highest income. Among outright owners, households with a very low income level and almost no savings choose to sell. Among CC or FRM holders, households with higher income keep their houses, and low-income households either sell or default on the debt. This choice depends on the household's equity level in the home. Both CC and FRM holders choose to default if

Figure 3: Fraction of active renters using a CC to all borrowers
This figure shows the ratio of active renters who use a CC for borrowing to active renters who take out a loan, either with a CC or FRM. When the difference between the cost of funding for CC-issuing and FRM-issuing lenders is more than 30 basis points, CCs are wiped out.



they are underwater, so their net wealth is negative.

Table 4 compares the mortgage premium and down payment in the benchmark and counterfactual economies. Compared to the benchmark economy, the average mortgage premium is higher in CF2, since CCs have a higher premium compared to FRMs. In CF1, the mortgage premium increases again but note that the premium is defined as the difference between the mortgage interest rate and the risk-free interest rate r_f. In CF1, the cost of capital is 45 basis points (τ) higher compared to CF2, and the increase in mortgage premium is much less than this amount, since borrowers respond to this change by increasing the down payment by a substantial amount. Down payments for CCs increase so much that the mortgage premium is constant for CCs in both CFs.

If the difference between the cost of capital for FRM and CC issuing lenders is set to τ, then CCs are wiped out from the market, as seen in Table 3. The estimate for the difference in cost of funding in the real estate literature varies, so I will show the robustness of my results for different values of τ. Figure 3 shows the fraction of active renters who buy a house with a CC to all borrowers, depending on the value of τ. Findings show that as long as the cost of the funding advantage for GSEs is more than 30 basis points, this would be enough to eliminate CCs from the market. The y-axis is equal to 0.62 when the cost advantage is set to 0, and this corresponds to the ratio of CC users to all borrowers in CF2 in Table 3: $6.2/(6.2 + 3.8) = 0.62$.

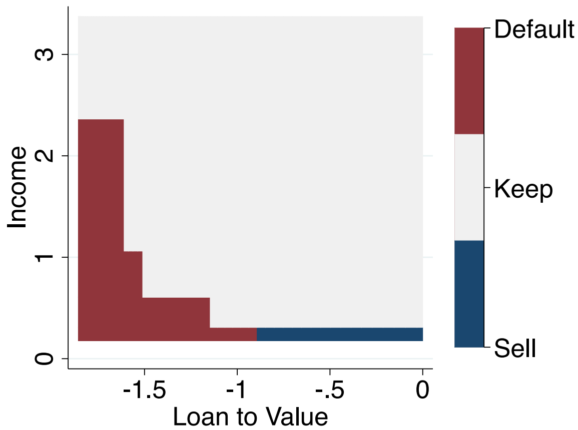
6.3 Mechanisms of the Model

In this subsection, I will show the policy function of a borrower to explain default dynamics and, based on the default decisions of borrowers, I will discuss the pricing of both contracts. Then, I will analyze how the emergence of CCs affects households over the life cycle. For my analysis with CCs, I will use CF1 in which I assume that there is no government intervention in the housing market; results for CF2 will be provided in the Appendix.

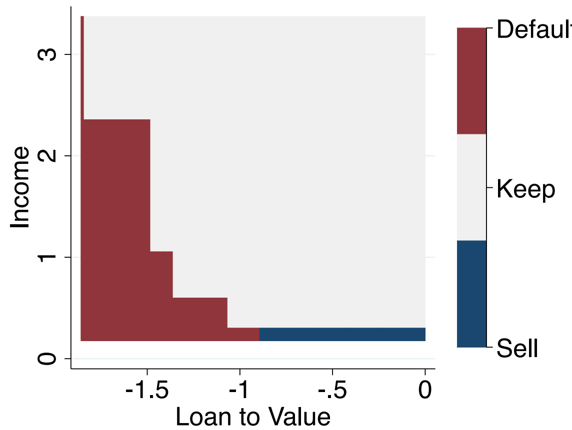
How CCs affect default probability and foreclosure rates can be seen in Figure 4 which shows the policy function of a homeowner holding a CC or an FRM with different interest rates. A loan holder has three choices: (i) make at least the periodic payment and keep the house, (ii) sell the house and pay back the debt, or (iii) default on the loan. Both figures show that in the case of a low aggregate house price shock, loan holders keep their house if their income level is high. If their income is low, they either sell it if the LTV ratio is low or default on the mortgage if the LTV ratio is high. This is consistent with empirical findings on mortgage default. Comparing the policy function of homeowners with CCs and FRMs shows that the likelihood of defaulting on the debt is lower for CCs, since payments are adjusted due to a low house price shock. On the other hand, an increase in the interest rate also leads to an increase in the likelihood of default. Given that interest rates are mostly higher for CCs, this can lead to higher default rates for CCs in equilibrium, even in an economy hit with an aggregate house price shock.

Figure 5 shows the mortgage premium—the mortgage interest rate minus the risk-free interest rate—for FRMs and CCs for different down payment and income levels. For all income levels and both contracts, the mortgage premium is higher for lower down payment levels because lower down payments are associated with a higher probability of default. Furthermore, a higher probability of default leads to a higher mortgage

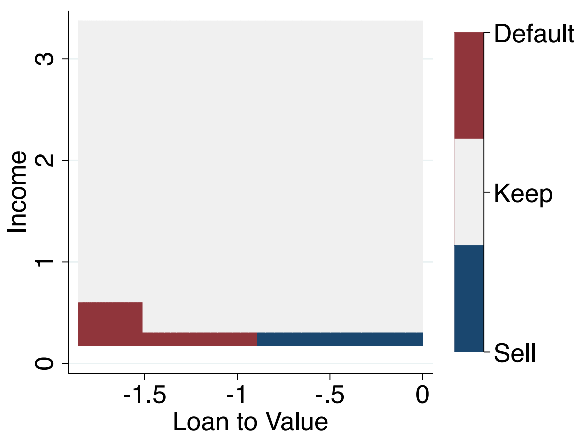
Figure 4: Default decision of a homeowner hit with an aggregate low house price shock
Figures show the policy function of a loan holder hit with a low aggregate house price shock for different contracts and different interest rates. Both the default and selling probabilities are lower for a CC holder, since periodic payments are adjusted downward in case of a low aggregate house price shock. Default probability increases with the mortgage interest rate.



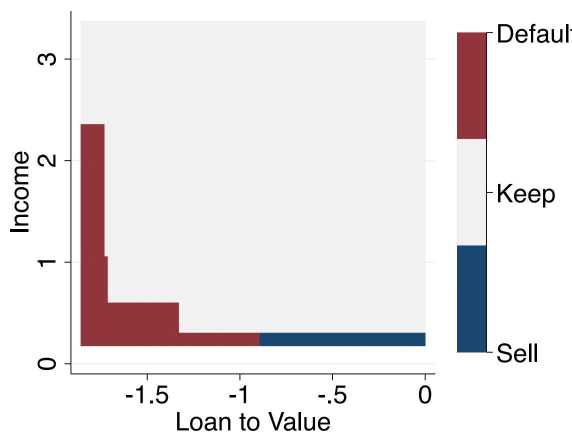
(a) FRM, low interest



(b) FRM, high interest



(c) CC, low interest

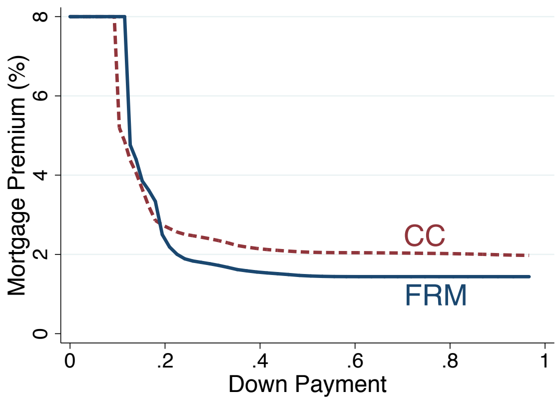


(d) CC, high interest

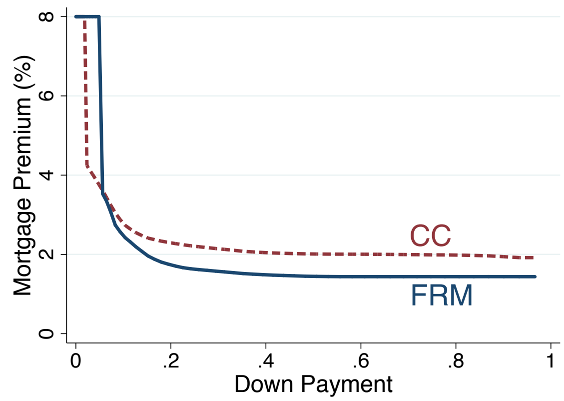
premium because foreclosures are costly for the lender due to two reasons: (i) the foreclosure cost θ_l that the lender has to pay, and (ii) foreclosures happen when the homeowner has negative equity.

Furthermore, mortgage premium for FRMs is lower for high down payment options but higher for low down payment options due to following mechanism. Mortgage premium converges to $\kappa + \tau$ for FRMs as the downpayment increases, since the probability of default converges to zero. For CCs, the mortgage premium converges to a higher

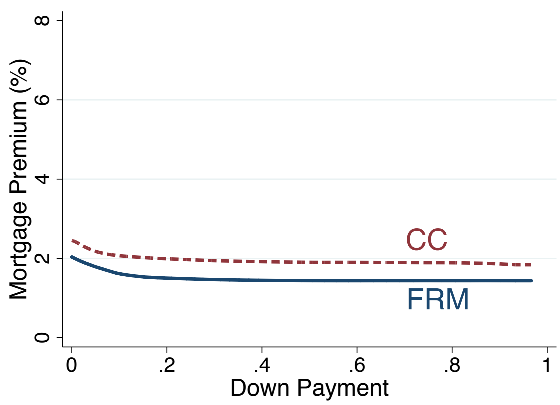
Figure 5: Mortgage premium for different income levels in CF1
This figure shows the mortgage premium at the steady state for 30 year maturity FRMs and CCs in a counterfactual setting where the cost of capital is the same and equal to $r_f + \tau$ for both contracts.



(a) low income



(b) median income

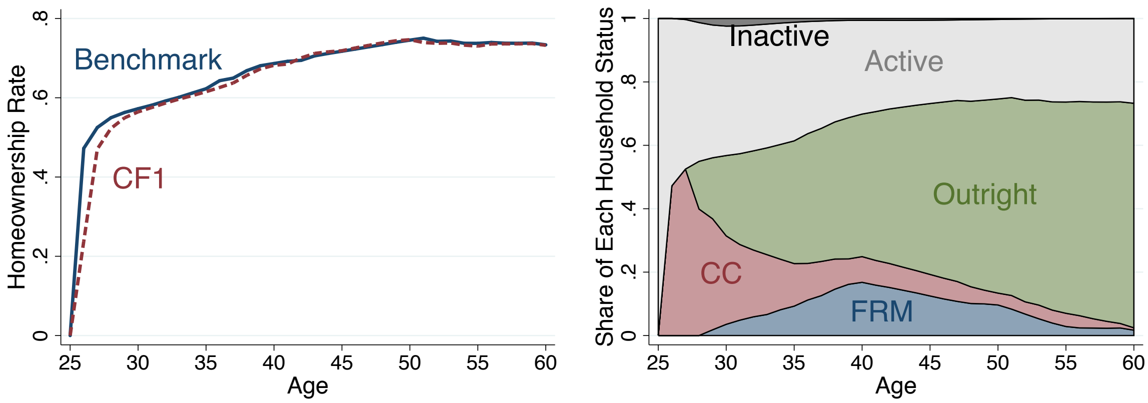


(c) high income

constant due to house price volatility, since CC payments are adjusted downward in case of an aggregate house price shock. Therefore, even if the default probability is zero for both contracts, the mortgage premium for CCs is higher than the premium for FRMs. On the other hand, for low down payment options, CCs decrease the probability of default relative to FRMs, and make it is less likely for the lender to incur the foreclosure costs. Therefore, as the down payment decrease, increase in the probability of default is relatively lower for CCs, so they have a lower mortgage premium.

To see the effect of foreclosure cost θ_l, Figure 11 shows the interest rate for both contracts when $\theta_l = \theta_h = 0$, so that neither the lender nor the homeowner pays any

Figure 6: Homeownership and contract choice over the life cycle
This figure shows the homeownership rate in the benchmark model and CF1 and the share of each household status over the life cycle.



(a) home ownership (b) contract choice in CF1

transaction cost to sell the house. In this case, mortgage premiums are lower. Similarly, the premium for both contracts is lower for higher down payments, and the premium for CCs converges to a higher level compared to FRMs. Note that there are still cases in which CCs have a lower premium, since the selling cost of foreclosed house θ_l is not the only cost of foreclosure: In equilibrium, default happens when the homeowner has negative equity, and therefore foreclosures are still costly for lenders.

In Table 3, we see that active renters who borrow have higher wealth on average, and Table 4 shows that borrowers make a higher down payment in CF1 compared to the benchmark economy. Also, the homeownership rate is slightly lower in CF1 relative to the benchmark economy. Consistent with these findings, in Figure 6 we see that the homeownership rate is slightly lower in CF1 for the beginning of the life cycle. In CF1, the cost of capital is higher for FRMs, and this leads to an increase in mortgage interest rates. Households respond to this change by increasing the down payment they make to decrease their periodic payments. As a result, in CF1 households starting their life cycle need to wait a bit longer to save enough to make a higher down payment. This leads to a delay in buying a house and is the main reason for the decrease in average homeownership rate, since homeownership rates are almost equal for later periods of life in both economies.

The right panel of Figure 6 shows the contract choices of households over the life cycle in CF1. Households that want to own a house borrow at earlier periods of their lives and then become outright owners after paying the mortgage debt. Most borrowers choose to use a CC at earlier periods, since the premium is lower for low down payment options with CCs (see Figure 5). This leads to a bias in borrower characteristics, such that most risky borrowers choose to borrow with CCs since they are less expensive for risky borrowers. Comparing active renters who borrow with CCs and FRMs in Table 3 in CF1 shows that borrowers using CCs have higher income and lower wealth on average. The wealth of CC users is 0.19, while it is 0.43 for FRM users.

6.4 Aggregate Shock

In this subsection, I will show the implications of CCs for an economy hit with a low house price shock. To conduct this analysis, I simulate the economy by setting the aggregate house price level to 1 for a long period, then apply aggregate house price shocks to the economy at the steady state. Figure 7 shows the foreclosure rate in the data, the benchmark economy, and CF1. In the benchmark economy, households use only FRMs but in CF1, the foreclosure rate is a weighted average of foreclosure rates for CCs and FRMs. Since I calibrate the model so that the steady state foreclosure rate in the benchmark economy is equal to the average foreclosure rate in the data for the period 1990-2000, foreclosure rates in the data and benchmark economy are very close to each other for that period. Consistent with the data, the aggregate house price shock hits at 2007. The benchmark model follows the foreclosure rate in the data very closely for earlier periods of the crisis, but households respond to shocks earlier in the model than in the data.

Table 5 shows the details in Figure 7. The foreclosure rate in the benchmark economy is 0.7% at the steady state and this is also the foreclosure rate among FRM holders. In CF1, average foreclosure rate is 0.5% which is a weighted average of foreclosure rates for CCs and FRMs, and foreclosure rate for CCs is higher. Although, this may seem surprising at first, note that contingent contracts are contingent on the aggregate house prices and borrowers do not benefit from the contingency at the steady state in which there is no aggregate shock. In other words, CCs are an insurance against aggregate house price shocks and at the steady state, borrowers do not enjoy the benefits of insurance but simply pay the premium of it.

Right panel of Table 5 shows the increase in foreclosure rates when a 30% aggregate

Figure 7: Foreclosure rate
This figure show the foreclosure rate in the data, benchmark model and CF1. Foreclosure rate in CF1 is a weighted average of foreclosure rate for CCs and FRMs.

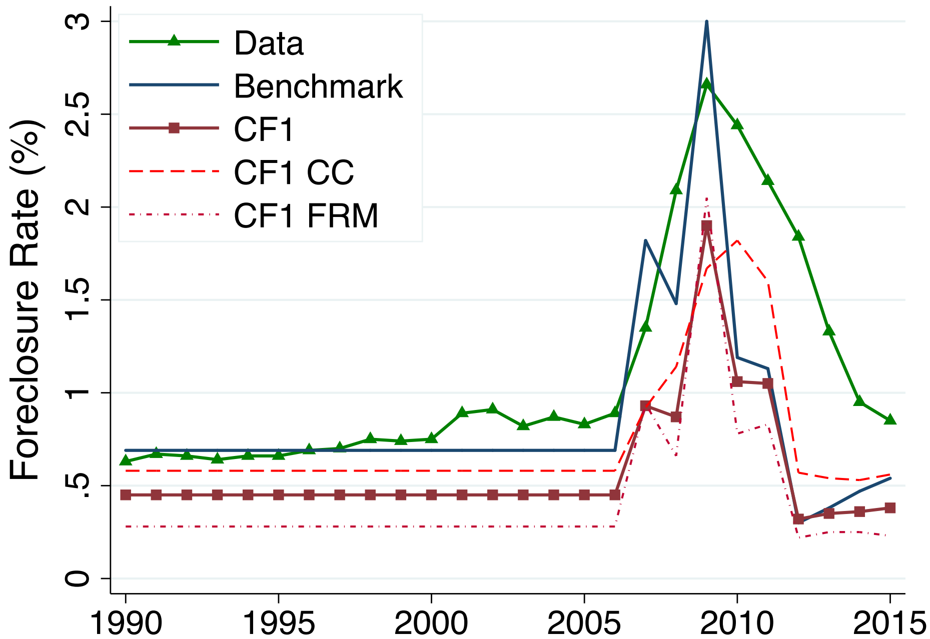


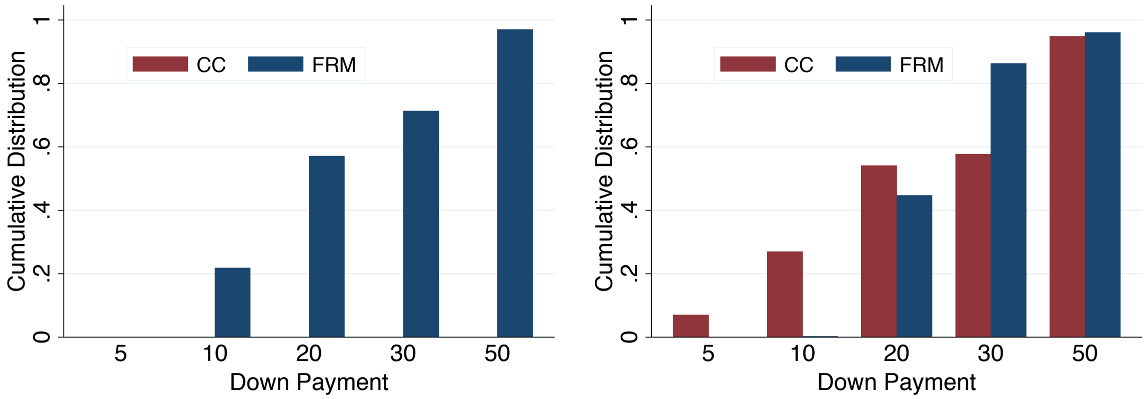
Table 5: Foreclosure rate at the steady state and after the aggregate shock
This table shows the foreclosure rate at the steady state and during the aggregate shocks for the benchmark economy and CF1. Red numbers are the increase in the foreclosure rates due to 30% aggregate house price shock. Note that since none of the borrowers choose to use CCs in the benchmark economy, foreclosure rate is 0 for contingent contracts.

	Steady State		Aggregate shock	
	Benchmark	CF1	Benchmark	CF1
Average	0.7	0.5	0.7+2.3=3.0	0.5+1.4=1.9
FRM	0.7	0.3	0.7+2.3=3.0	0.3+1.8=2.1
CC	0	0.6	0	0.6+1.1=1.7

house price shocks hit this economy. The foreclosure rates increase by 2.3%, 1.4%, 1.8% and 1.1% for the benchmark economy, CF1, FRMs in CF1 and CCs in CF1, respectively. Increase in foreclosure rate for CCs in CF1 is the lowest among others and this is reasonable since CCs adjust the value of debt when the aggregate house price shock hits. Since the steady state value of foreclosure rate for CCs is 0.6%, this leads to a foreclosure rate of 1.7% for CCs when the shock hits.

On the other hand, foreclosure rate in CF2 is higher compared to the benchmark

Figure 8: Cumulative distribution of down payment for both contracts in the benchmark economy and CF1



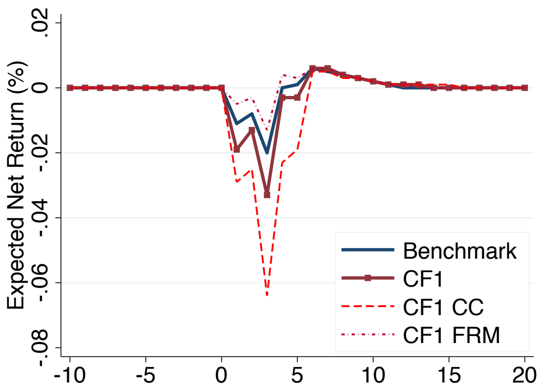
(a) Benchmark

(b) CF1

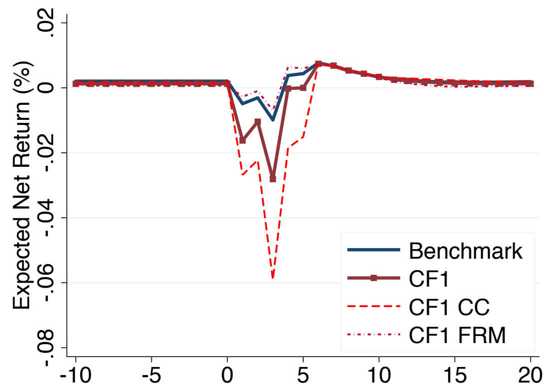
economy (see Table 7). Consistent with CF1, in CF2, foreclosure rate of CCs is higher than FRMs at the steady state and also when the aggregate house price shock hits, increase in foreclosure rate is lowest for CCs. However in CF2, steady state foreclosure rate of CCs is so high that the average foreclosure rate is higher than the benchmark economy.

Comparing the benchmark economy with CF1 or CF2 consistently shows that CCs has a higher foreclosure rate at the steady state but leads to a lower increase in the foreclosure rate in the event of an aggregate house price shock. In other words, CCs decrease the cyclicality of foreclosure rate but increase the foreclosure rate in normal times. This raises the following question: Why CCs have a higher foreclosure rate at the steady state? The underlying reason is in Figure 8. Although the average down payment ratio is higher in CF1 compared to the benchmark economy, share of households with very low down payments increase in CF1. In the benchmark economy, individuals who purchase a house with less than 5% down payment is almost negligible, while in CF1, around 7% of CC users choose to purchase a house with less than 5% down payment. Also, households borrowing with low down payments mostly choose to use CCs rather than FRMs, since the price of CCs is lower for low down payment options. This leads to a higher foreclosure rate at the steady state, in which the households are subject to idiosyncratic house price shocks. Note that when the house price of a borrower decreases due to an idiosyncratic house price shock, mortgage payments are not adjusted, since CCs are linked to aggregate house prices. In other words, removing

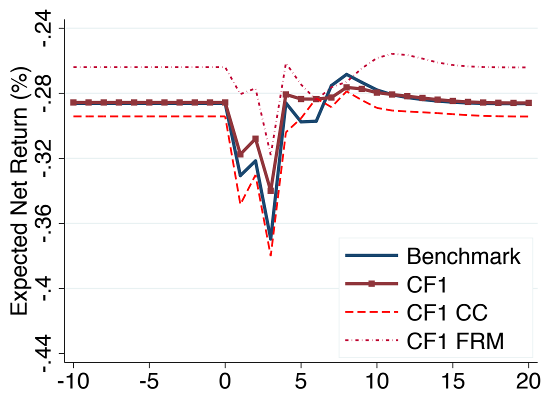
Figure 9: Expected net return of debt (%)
This figure shows the expected net return of debt for the benchmark model and CF1. The return in CF1 is a weighted average of CCs and FRMs. Also, the return for all borrowers is a weighted average of borrowers who choose to sell the house, keep the house or default. The return for sellers is always equal to 0 since they just pay back the debt to lender.



(a) All borrowers



(b) Borrowers keeping the loan



(c) Borrowers defaulting

the effect of government makes CCs relatively cheaper for low down payment options, and households respond to these prices by borrowing with low down payment CCs.

Another concern regarding CCs is their effect on the lender's balance sheet since CCs can create volatilities for the lender due to being contingent on the aggregate house prices. To address this concern, in Figure 9, I show the expected net return of debt when the economy is hit with an aggregate house price shock. Figure 9a shows the net return for all borrowers in the benchmark economy, CF1 and CC or FRM holders in CF1. Steady state value is equal to 0 due to zero profit condition of the lender. At the steady state, expected return of debt is equal to opportunity cost of debt so the net return is always equal to 0. Expected return for all borrowers is a weighted average of the expected return for borrowers who choose to sell the house, keep the house and default on the debt. If borrower sells the house, she pays back her debt to the lender, thus expected net return is always equal to 0 for sellers even when the economy is hit with an aggregate shock for all contracts and in all economies.

Table 6: Loss of lender (%) when hit with 30% aggregate house price shock
This table show the loss of the lender (%) when hit with a 30% aggregate house price shock. For instance, in the benchmark economy, average loss of the lender from all borrower is 2% or return of debt is equal to -2%.

	All borrowers	Borrowers Keep	Borrowers Default
Benchmark	2.0	1.0	37
CF1	3.3	2.8	34
CF1 CC	6.4	5.9	38
CF1 FRM	1.3	0.6	31

Figure 9b shows the expected return of the lender for the borrowers who choose to keep the house by making at least the periodic payments of the loan. Expected return at the steady state is slightly positive to compensate the lender for the losses due to foreclosures but since foreclosure probabilities are very small, expected return is very close to zero. Figure 9c shows the expected return of the lender for the borrowers who choose to default. Since, the foreclosure cost $\theta_l = 0.27$, expected return for the lender is around -27% for this subgroup. Note that steady state values of expected return for defaulters is consistent with Table 3. In this table, we see that the average wealth of defaulting household is equal to -0.11 in the benchmark economy and -0.14 and -0.02 in CF1 for CCs and FRMs respectively. Consistently in Figure 9, losses are highest for CC holders in CF1 and lowest for FRM holders in CF1. The weighted average of subfigure (b) and (c) together with the expected return for sellers constitute subfigure (a). In the case of an aggregate house price shock, expected return of the lender decrease and lender make negative profits during these periods. This is why I assume a deep pocketed lender since losses in these periods are compensated in later periods and the lender makes zero profit on average in the long run.

Volatility of the lender return increases due to payment adjustments in CCs. However, this is counterbalanced by the lower number of defaults as seen in Table 6 which shows the percent losses of the lender in the event of an aggregate house price shock.

First, even though the house prices and hence the periodic payments of CCs decrease by 30% due to aggregate house price shock, expected return of CCs for households keeping their loan decrease by just 5.9% since aggregate shocks are mean reverting. Second, in CF1 expected loss is 3.3% for all borrowers and 2.8% for households keeping their loans, thus defaulting households increase the loss of the lender by just 0.5%. On the other hand, in the benchmark economy, while expected loss for borrowers keeping their loan is 1%, it is 2% for all borrowers in the benchmark economy. So, defaulting households increase the loss of the lender by 1% since the share of defaulters is higher in the benchmark economy. Thus, regarding the lender losses in case of an aggregate house price shock, benchmark economy suffers from the high number of defaults and bigger losses due to defaults, while CF1 suffers from the decreasing periodic payments.

7 Conclusion

Empirical studies show that negative equity is a key trigger of default, and the main feature of mortgage contracts that are contingent on house prices is to prevent borrowers from falling into negative equity and, in turn, decrease the probability of default. The benefits of contingent contracts have been studied in the literature, and government intervention in the housing market is often argued to be the primary reason for the nonexistence of contingent contracts in practice. However, no quantitative analysis has been conducted to support this claim. My paper's first contribution is to quantify the effect of government intervention in an equilibrium model of housing and show that government intervention through government sponsored enterprises is sufficient to explain the nonexistence of contingent contracts.

Second, most papers in the related literature focus on the pricing of these contracts while ignoring the interplay between mortgage interest rates and borrower behavior; only a few recent studies analyze the effects of contingent contracts in an equilibrium model. I extend this literature by studying the implications of contingent contracts in an equilibrium model of housing with heterogeneous agents, and show that contingent contracts, as expected, decrease the cyclicality of foreclosure rate, but, notably, increase the average foreclosure rate at the steady state due to the following mechanism.

Contingent contracts decrease the probability of default by adjusting the value of debt in the event of an aggregate house price shock. So, they are cheaper than FRMs for low down payment options since foreclosures are costly for the lender. As a result, younger borrowers with low asset positions typically choose to borrow with low down payment contingent contracts, since they do not have much assets. At the steady state with just idiosyncratic house price shocks, contracts' contingency are not triggered and therefore the foreclosure rate gets higher, since contingent contract holders have less home equity. In other words, contingent contracts increase the average number of foreclosures in the economy by endogenously decreasing the down payment of households, since contingent contracts are relatively cheaper for low down payment options.

In the event of an aggregate house price shock, contingent contracts adjust the value of debt, and cause a mild increase in the number of foreclosures. While the foreclosure rate increases by 2.3% in the benchmark economy, and 2.0% in the data, it increases by 1.4% in the counterfactual world with contingent contracts. In terms of the value of debt for the lender, as argued in the literature, contingent contracts increase the volatility of lender return. This is counterbalanced, however, by the lower number of foreclosures relative to FRMs.

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8 APPENDIX

8.1 Computational Appendix

I discretize the state space for households for each state variable. The level of assets can take 60 values between 0 and $a_{max} = 30$. Grid points are unevenly spaced, so that they are denser close to zero. Average income $\bar{y}$ is normalized to 1, and the maximum level of assets is $30\bar{y}$ in the economy. Similarly, debt level and down payment ratio can also take 60 values. The grid for the down payment ratio can take values between 0 and 1, and is denser close to 0. The grid for debt level can take values between 0 and $\bar{p_h} \max_s(q_s)$, and is denser close to the maximum debt level. Mortgage interest rates take evenly spaced 30 values between $r_f = 4\%$ and 12%, which is the maximum value that r_m can take. Idiosyncratic house prices and income shocks are discretized using Adda Cooper method with 5 states. Aggregate house prices can also take 5 values. Uneven spacing between grid points for some variables is done to create more grid points in the regions in which utility function or value functions have higher curvature. Then, I follow the steps below:

1. I start to solve the the problem of households at age J, the last period before retirement. There is no borrowing at age J so I do not need mortgage prices at that period. Also, the retired household's problem is deterministic and can be solved analytically.

2. Then, after finding the policy functions of households at age J, I can find the zero profit mortgage interest rates for an individual at age $J-1$. After finding the mortgage rates, I can also find the policy functions at age $J-1$, too. So, from age $J-1$ to age 1, I first solve the mortgage prices at age $j \in 1, ..., J-1$ given the policy functions at age $j+1$, then I solve the policy functions at age j given the mortgage prices at age j. The details are as following:

   a) Consider an active renter who is considering borrowing to buy a house at age $j \leq J-1$. I know her asset and income level, and also the aggregate state of the economy. She has to choose between a CC and an FRM, and for each contract type there are 60 downpayment options she can choose. So, the borrower has $2 \times 60 = 120$ options in terms of borrowing. Then, for each of these options, the lender can set 30 different mortgage interest rates and for each of these interest rates there is a corresponding level of debt for the next period. In the next period, for each interest rate and level of debt, there are ($5 \times 5 \times 5 = 125$) states of the world corresponding to different income levels, idiosyncratic house price levels and aggregate states. Since I know the policy function of the household at age $j+1$ for all these states of the world, l can compute the expected return of debt for each of 30 different mortgage interest rates. After obtaining a mapping from mortgage interest rates to expected return of debt, using *zbrent* I find the mortgage interest rate so that the expected return is equal to the cost of capital.

b) After finding the mortgage interest rate for each $2 \times 60 = 120$ options for borrowers as above, I solve the household's problem at age j. Continuation values for existing debt are found in a similar way.

3. In the first period, I solve only the problem of an active renter given the mortgage prices; I do not need policy functions for other types, since households start their life cycle as active renters.

4. Before I simulate the economy using policy functions, I find the policy functions over a denser state space using linear interpolation. I use 200 grid points for asset and debt level so that the distribution of households becomes smoother. I do not solve the policy functions over this denser state space initially to decrease the computational burden.

5. After finding the policy functions on the much denser state space, I pick an initial distribution for the population. Initially, all households have zero wealth, and distribution of households over the state space of income shocks is equal to steady state distribution of the Markov chain for income process. Then I fix the aggregate state of the economy so that $q_s = 1$.After a long period of simulation so that the economy reaches to steady state, I compute the statistics of interest.

6. To estimate the jointly selected parameters, probability of being an active renter δ, utility premium of homeownership ϕ_h, and service cost κ, I minimize the sum

of squared deviations of target statistics from data counterparts.

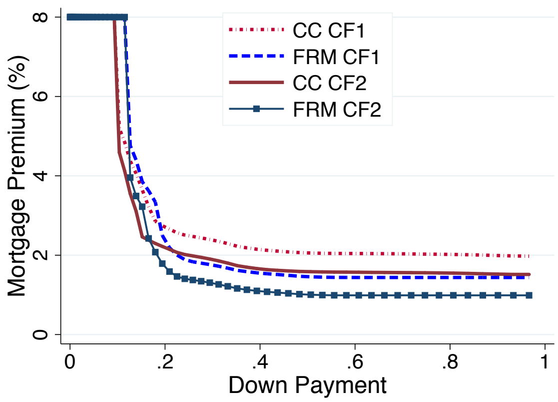
$$\min_{\beta,\phi_h,\kappa} \sum_{i=1}^{3} \left(\frac{m_i(\beta, \phi_h, \kappa) - m_i(data)}{m_i(data)} \right)^2$$

where (m_1, m_2, m_3) =(homeownership rate, foreclosure rate, mortgage premium).

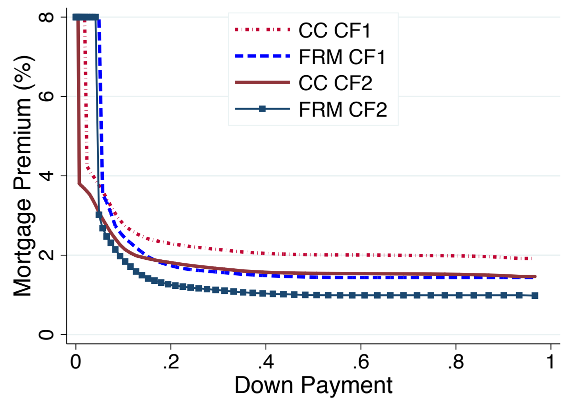
7. For the minimization of the objective function, I use a global optimization algorithm. At each iteration, I start a local optimization at a different parameter set using a quasi random Sobol sequence. For local optimization, I use Nelder-Mead (amoeba) algorithm.

8.2 Figures and Tables

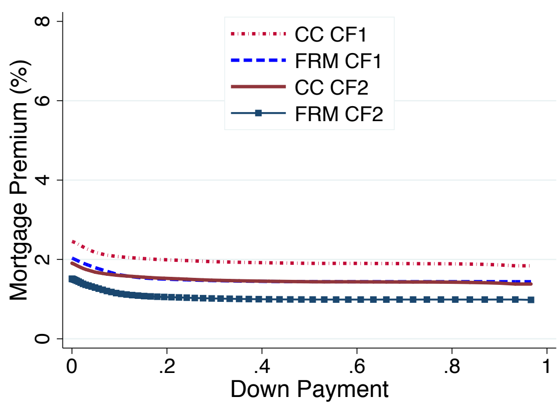
Figure 10: Mortgage premium for different income levels in CF1 and CF2
This figure shows the mortgage premium at the state state for 30-year maturity FRMs and CCs in CF1 and CF2. In CF1, cost of capital is τ bp higher for both contracts.



(a) low income

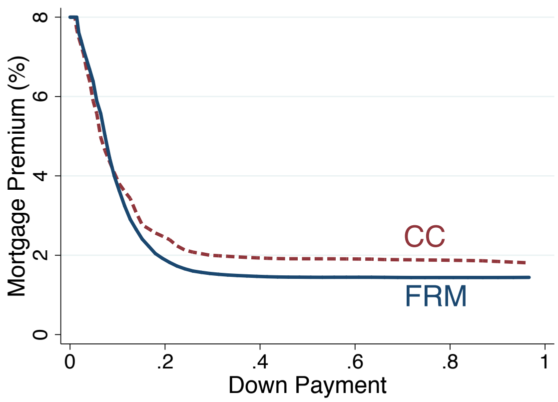


(b) median income

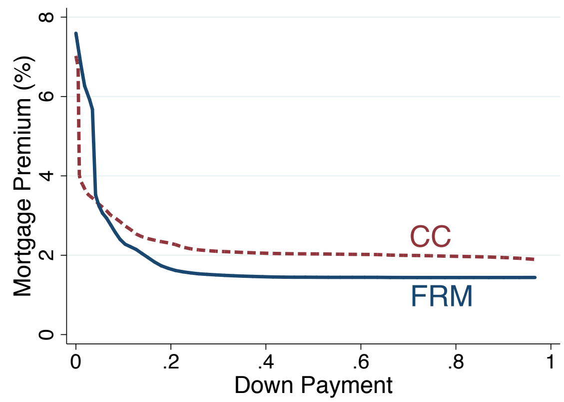


(c) high income

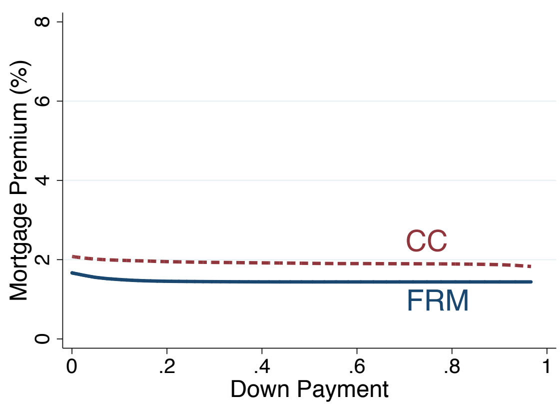
Figure 11: Mortgage premium for different income levels when $\theta_l = \theta_h = 0$.
This figure shows the mortgage premium at the steady state for 30 year maturity FRMs and CCs in a setting where the cost of capital is the same and equal to $r_f + \tau$ for both contracts and also $\theta_l = \theta_h = 0$. This figure shows that even though the transactions costs are equal to zero, defaults are still costly for the lender, since in equilibrium, homeowners default when they are underwater.



(a) low income



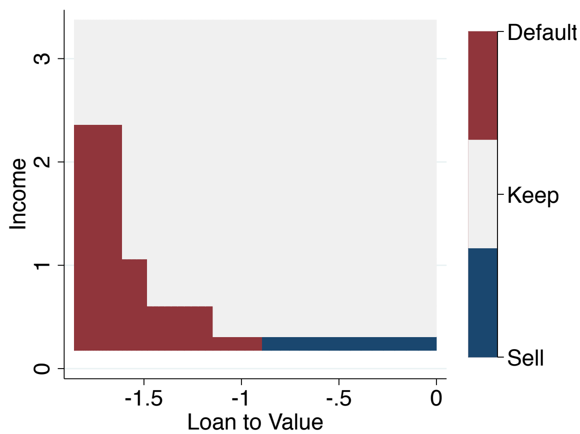
(b) median income



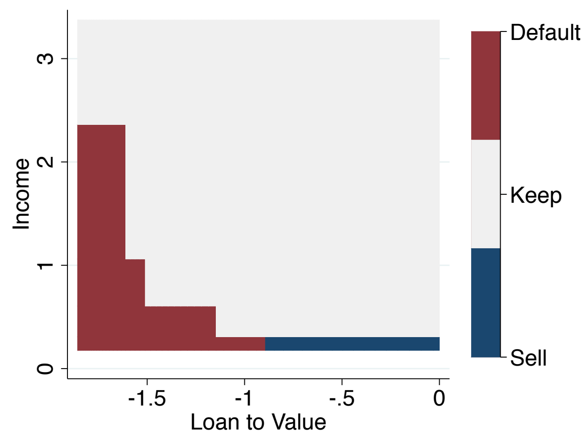
(c) high income

Figure 12: Policy function of a homeowner hit with a low aggregate house price shock in the benchmark economy and counterfactuals

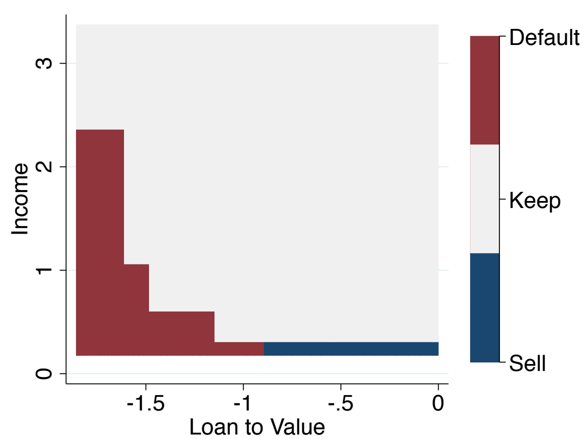
This figure shows the policy function of a homeowner. A homeowner can choose to sell the house, keep the house by paying at least the periodic payment, or default on the debt. Each subfigure shows the policy function for an identical household in different economics, so policy functions are almost the same. The default probability of a household is slightly lower in CF1, but the same in the benchmark economy and CF2.



(a) Benchmark



(b) CF1



(c) CF2

Figure 13: Foreclosure rate in CF2

This figure show the foreclosure rate in the data, benchmark model and CF2. Foreclosure rate in CF2 is a weighted average of foreclosure rate for CCs and FRMs. Comparing with CF1, steady state foreclosure rate is higher but the increase in the foreclosure rate during the aggregate shock periods is lower. Thus, this graph also verifies that there is a trade of between steady state foreclosure rate and the foreclosure rate during crisis.

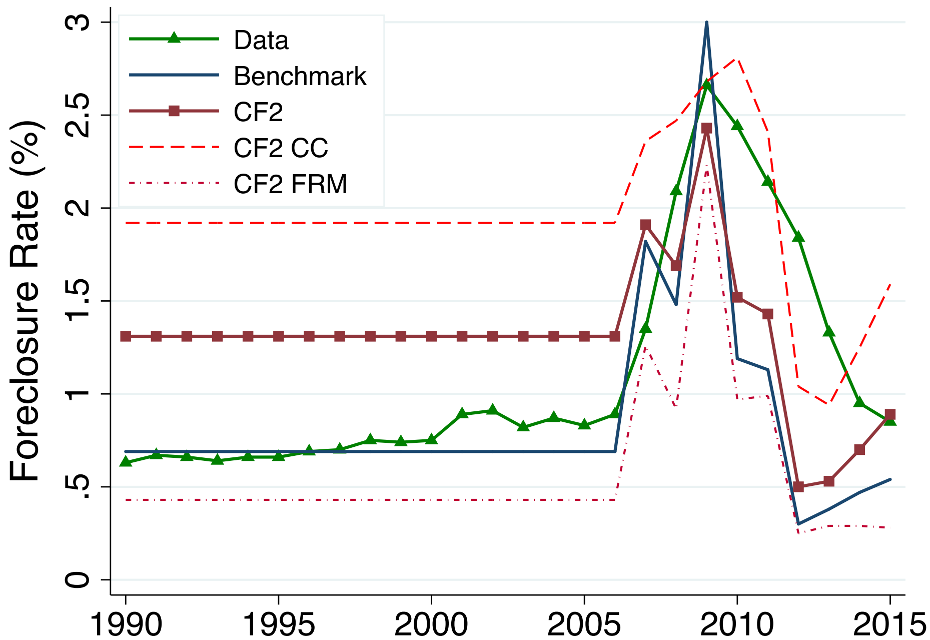
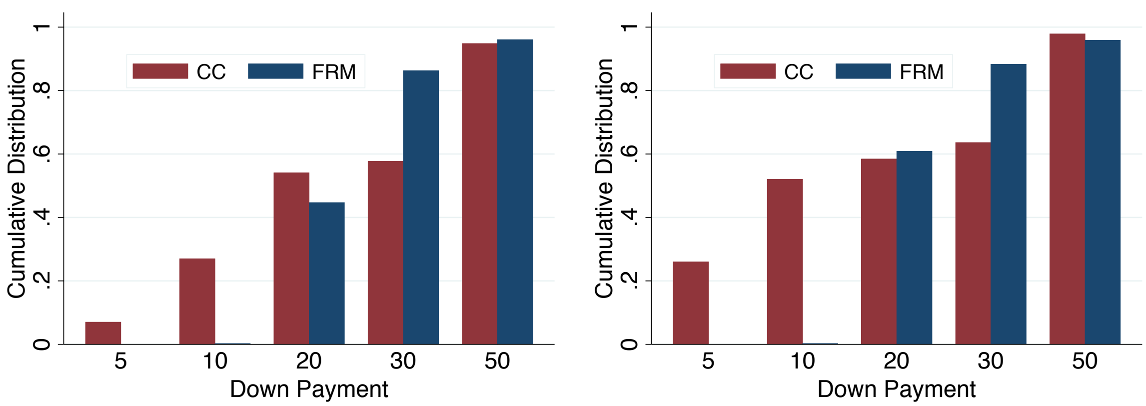


Table 7: Foreclosure rate at the steady state and after the aggregate shock

This table show the foreclosure rate at the steady state and during the aggregate shocks for the benchmark economy and CF2. Red numbers are the increase in the foreclosure rates due to aggregate house price shock. Note that since none of the borrowers choose to use CCs in the benchmark economy, foreclosure rate is 0 for contingent contracts.

	Steady State		Aggregate shock	
	Benchmark	CF2	Benchmark	CF2
Average	0.7	1.3	0.7+2.3=3.0	1.3+1.1=2.4
FRM	0.7	0.4	0.7+2.3=3.0	0.4+1.8=2.2
CC	0	1.9	0	1.9+0.8=2.7

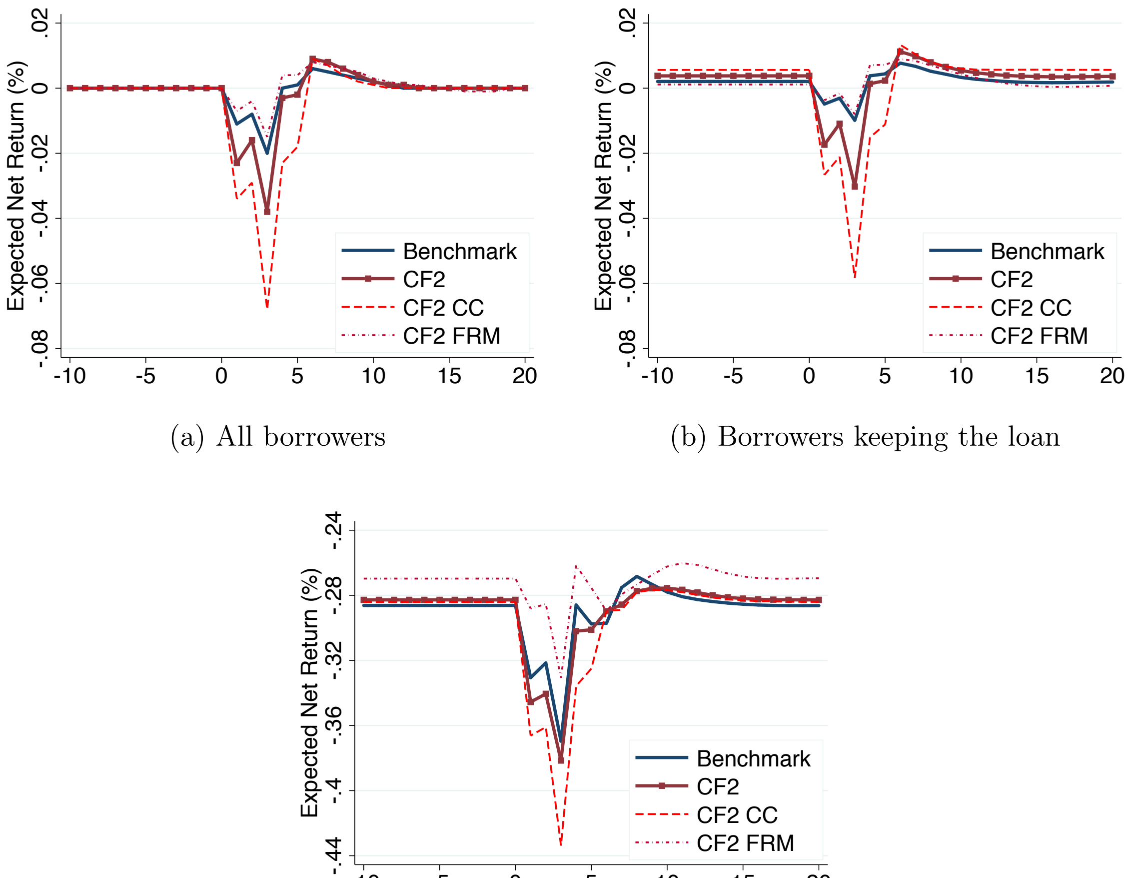
Figure 14: Cumulative distribution of down payment for both contracts in CF1 and CF2
This figure shows that borrowers using CCs with less than 5% downpayment increase in CF2 compared to CF1, responding to change in mortgage interest rates.



(a) CF1 (b) CF2

Figure 15: Expected net return of debt (%)

This figure shows the expected net return of debt for the benchmark model and CF2. The return in CF2 is a weighted average of CCs and FRMs. Also, the return for all borrowers is a weighted average of borrowers who choose to sell the house, keep the house or default. The return for sellers is always equal to 0 since they just pay back the debt to lender.



(a) All borrowers

(b) Borrowers keeping the loan

(c) Borrowers defaulting